

NAG Toolbox for MATLAB

f07te

1 Purpose

f07te solves a real triangular system of linear equations with multiple right-hand sides, $AX = B$ or $A^T X = B$.

2 Syntax

```
[b, info] = f07te(uplo, trans, diag, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07te solves a real triangular system of linear equations $AX = B$ or $A^T X = B$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J 1989 The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **trans** – string

Indicates the form of the equations.

trans = 'N'

The equations are of the form $AX = B$.

trans = 'T' or 'C'

The equations are of the form $A^T X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3: **diag** – string

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n triangular matrix A .

If **uplo** = 'U', A is upper triangular and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', A is lower triangular and the elements of the array above the diagonal are not referenced.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

5: **b(ldb,*)** – double array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$

The n by r right-hand side matrix B .

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The second dimension of the array **a**.

n , the order of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p** – int32 scalar

Default: The second dimension of the array **b**.

r , the number of right-hand sides.

Constraint: $\mathbf{nrhs_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

5.4 Output Parameters

1: **b(ldb,*)** – double array

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs_p})$

The n by r solution matrix X .

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **trans**, 3: **diag**, 4: **n**, 5: **nrhs_p**, 6: **a**, 7: **lda**, 8: **b**, 9: **ldb**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , $a(i, i)$ is exactly zero; A is singular and the solution has not been computed.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham 1989.

For each right-hand side vector b , the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon|A|,$$

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_{\infty}}{\|x\|_{\infty}} \leq c(n) \text{cond}(A, x)\epsilon, \quad \text{provided} \quad c(n) \text{cond}(A, x)\epsilon < 1,$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_{\infty} / \|x\|_{\infty}$.

Note that $\text{cond}(A, x) \leq \text{cond}(A) = \| |A^{-1}| |A| \|_{\infty} \leq \kappa_{\infty}(A)$; $\text{cond}(A, x)$ can be much smaller than $\text{cond}(A)$ and it is also possible for $\text{cond}(A^T)$ to be much larger (or smaller) than $\text{cond}(A)$.

Forward and backward error bounds can be computed by calling f07th, and an estimate for $\kappa_{\infty}(A)$ can be obtained by calling f07tg with **norm_p** = 'I'.

8 Further Comments

The total number of floating-point operations is approximately n^2r .

The complex analogue of this function is f07ts.

9 Example

```
uplo = 'L';
trans = 'N';
diag = 'N';
a = [4.3, 0, 0, 0;
     -3.96, -4.87, 0, 0;
     0.4, 0.31, -8.02, 0;
     -0.27, 0.07, -5.95, 0.12];
b = [-12.9, -21.5;
     16.75, 14.93;
     -17.55, 6.33;
     -11.04, 8.09];
[bOut, info] = f07te(uplo, trans, diag, a, b)
```

```
bOut =  
  -3.0000  -5.0000  
  -1.0000   1.0000  
   2.0000  -1.0000  
   1.0000   6.0000  
info =  
      0
```
